Prediction of Cutting Force in 3-Axis CNC Milling Machines Based on Voxelization Framework for Digital Manufacturing

Omid Yousefian, Joshua A. Tarbutton
University of South Carolina, Columbia, SC, USA
Department of Mechanical Engineering
omid@email.sc.edu, jat@sc.edu

Abstract
A new digital-based model is presented for the prediction cutting forces in 3-axis CNC milling of surfaces. The model uses an algorithm to detect the work-piece/cutter intersection domain automatically for given cutter location (CL), cutter and work-piece geometries. The algorithm uses a voxelization framework to voxelize the cutter helix as well as the work-piece and based on voxel intersections between them can detect the tool engagement. Furthermore, an analytical approach is used to calculate the cutting forces based on the discretized model. The results of model validation experiments on machining PMMA and Aluminum 6061 are also reported. Comparisons of the predicted and measured forces show that this digital approach can be used to accurately predict forces during machining.

Keywords: cutting force prediction, digital manufacturing, voxelization framework

1 Introduction
3-Axis CNC milling machines are widely used in industry to produce parts from a variety of materials with a variety of cutting tools. Based on the part function and geometry, different cutting tools are employed during milling. Simple flat end mills are used in peripheral milling of prismatic
parts, while straight and helical ball end mills are widely used in machining sculptured die and aerospace part surfaces. Ball nosed cutters are used to produce a plethora of parts where fillets are needed. Tapered end mills are used in five axis machining of jet engine compressors, and form cutters are used to open complex profiles such as turbine blade carrier rings (Engin & Altintas, 1999). To generate G-Code for a specific machine tool, the user is required to provide all of the input parameters needed such as: cutter geometry and material, desired depth of cut, spindle speed, and feedrate. These inputs are often based on handbooks and years of experience. Due to infinite number of parameters that affect the quality of the final part, the user usually chooses the “appropriate” parameters to get a path and machine the part. Process engineers do not have the luxury of evaluating the tradeoffs between tool geometries and various other process parameters. In addition, every tool path iteration is time consuming. The input parameters for machining should be a function of the features of the part instead of a function of what tools are on hand or what has worked in the past. The ability to automate this process while considering as many parameters as possible in order to generate more optimal tool paths will significantly increase manufacturing productivity. Predicting the cutting forces during the milling process can help engineers choose the “appropriate” cutting parameters such as: radial and axial depth of cut, type of tool, spindle speed and feedrate. Simulating the cutting forces in an appropriate digital framework plays an important role in automating the process of tool path generation because this simulation leads to determining the path parameters. In addition to affecting wear and breakage of the tool, which play an important role in cost of the finished part, predicting the cutting force helps users improve the surface quality and reduce machining damage. Lee and Altintas developed a model for predicting cutting forces in ball end milling based on the orthogonal cutting data (Lee & Altintas, 1996). Zeroudi et al. described a new approach for direct prediction of cutting forces in 3-axis milling machines from the tool path provided by CAM packages (Zeroudi, Fontaine, & Necib, 2010). Wei et al. used Z-level contouring tool path approach to predict cutting force in 3-axis CNC machines during machining of sculptured surfaces by ball-end tools (Wei, Wang, Zhu, & Gu, 2011). Wang et al. established an analytical cutting force model for helical milling (Wang, Qin, Ren, & Wang, 2012). In recent studies of cutting force prediction models through digital frameworks, Wou et al. developed a model for predicting the ball-end cutting forces in digital framework on a voxel-based and ray casting technique (Wou, Shin, & El-Mounayri, 2013). In their study, instantaneous uncut chip thickness is calculated based on the voxelized work-piece and ray-casted method. Through their model, the cutting pressure coefficients and cutting force prediction model can be extracted. Boz et al. introduced a new approach for detecting the engagement region between tool and work-piece. They employed this approach for predicting the cutting forces in five-axis milling of sculptured surfaces by ball-end mill (Boz, Erdim, & Lazoglu, 2011). Fussell et al. used an extended Z-buffer model to represent the work-piece geometry and a discrete axial slice model to represent the cutter. They used swept envelope of the toolpath to approximate the intersection of tool envelope with Z-buffer elements and consequently to find the engagement region for tool and work-piece (Fussell, Terard, & Hemmert, 2003).

Most of the models mentioned above are based on the specific geometry of the tool and work-piece or specific type of milling and in most of the cases; they are unable to define a generalized model which can predict the cutting forces for any arbitrary tool geometry and work-piece. Furthermore, the previous models for predicting the forces cannot be easily adopted for parallel computing framework. Tarbutton et al. described a new model based on voxelization and ray-casting of the part model (Tarbutton J. A., Kurfess, Tucker, & Konobrytskiy, 2013) which was employed to define a graphic-based approach for tool path planning (Tarbutton J. A., Kurfess, Tucker, & Konobrytskiy, 2013). Due to the high compatibility of their model to parallel computing framework their voxelization and ray-casting approach can be easily run on GPU in order to significantly decrease the computational time. In this paper, a voxel based model is selected to represent the blank and part during simulation of the machining process. In a voxel model, the three dimensional space consists of atomic cells called voxels. The main advantage of this approach is that most of the calculations are not
performed with floating point numbers as for geometric modeling kernels, but integers which are usually much faster on modern hardware. Moreover, through using a discretized framework like voxel model, engagement region between tool and work-piece can be easily detected without necessitating complex mathematical models.

In this paper, first the voxelization framework and geometry of cutting is studied. Then the cutting force model is explained and in the last step the simulation results are evaluated with the results from experiment.

2 Voxel Model and Geometry of the Cutting

In this section the voxel model, developed for the geometry of the cutting is explained. A voxel is considered as the smallest cubic volume element of a three-dimensional volume. For simulating the work-piece each voxel is classified into 3 categories: part voxel, blank voxel and empty voxel. A part voxel is a voxel that is inside the main part and should be kept when the milling is finished. A blank voxel is a voxel that belongs to the blank but not to the work-piece and an empty voxel is the space representing the volume after milling. For simulating the cutting tool each voxel is classified into 2 categories: cutting voxels and non-cutting voxels. Cutting voxels are the ones located on the cutting helix of the tool and cause the cutting process. On the other hand, non-cutting voxels are the ones located in the tool shaft but do not engage in cutting and merely represent the limits of the tool as a solid shape. The whole milling process can be defined as changing blank voxels to the empty ones by the cutting voxels of the tool. The physics of the problem demonstrates that in the voxelization framework the cutting force during any cutting process is merely applied on the cutting voxels. Thus, in the calculations and engagement detections, only the cutting voxels are considered which greatly reduces the computational cost. Figure 1 shows the voxelized volume of the part and blank. Red and blue volumes in the figure represent the part and blank respectively.

![Figure 1: Voxel representation of part (red) and blank (blue) volume.](image)

In order to voxelize the generalized tool, first the geometry of the tool should be studied. A generalized tool geometry with its parameters is shown in Figure 2 (Engin & Altintas, 1999). Among all the parameters for the generalized tool, only \( R, R_r, R_c, D, h, \alpha, \beta \) are independent parameters for characterizing the tool geometry. These are the seven geometric parameters that APT and CAD/CAM packages use to define the tool envelope.

Based on the above parameters, the radius of the tool at each specific \( z \) can be calculated by the following equations and subsequently the non-cutting voxels are determined (Engin & Altintas, 1999) based on the circle equation with \( r(z) \) as the radius:
For zone OM
\[
\begin{align*}
r(z) &= \frac{z}{\tan \alpha} \\
r(z) &= \sqrt{R^2 - (R - z)^2} + R_r \\
r(z) &= u + z \tan \beta, \quad u = \frac{D}{2} \left(1 - \tan \alpha \tan \beta\right)
\end{align*}
\]

For zone MN

For zone NS

The helix equation for generalized cutter is calculated as follows to determine the cutting voxels:

\[
\begin{cases}
x = R_t \times \cos \left(\frac{t}{a} + i \times \varphi_p\right) & 0 < t < L \\
y = R_t \times \sin \left(\frac{t}{a} + i \times \varphi_p\right) & i = 1, 2, \ldots, N (\text{number of the flutes}) \\
z = t
\end{cases}
\]

where \(L\) is the cutting length of the cutter and \(R_t\) is the radius of the cutter at each \(t\). \(\varphi_p\) is the angle between flutes and is calculated by the following equation:

\[
\varphi_p = \frac{2\pi \text{Number of flutes}}{2\pi}
\]

If \(\alpha\) is defined as the helix angle \(\alpha\) can be calculated as follows:

\[
\alpha = \frac{R_t}{\tan(\alpha)}
\]

Red and blue volumes in the figures of the Table 1 show the cutting and non-cutting voxels of the tool, respectively.
### Cylindrical End Mill

\[ D=0, R=0, R_w=D/2 \\
R_s=0, \alpha=0, \beta=0, h=0 \]

### Ball End Mill

\[ D=0, R=0, R_w=D/2, R_s=0 \\
\alpha=0, \beta=0, h=0 \]

### Ball Nose End Mill

\[ D=0, R=R_w=D/4 \\
\alpha=0, \beta=0, h=0 \]

### Taper End Mill

\[ D=0, R=0, R_w=D/2 \\
R_s=0, \alpha=0, \beta=0, h=0 \]

### Taper Ball End Mill

\[ D=0, R=R_w=0, R_s=0 \\
\alpha=0, \beta=0, h=0 \]

### General End Mill

\[ D=0, R=0, R_w=0, R_s=0 \\
\alpha=0, \beta=0, h=0 \]

---

**Table 1:** The Voxelization of different tool geometries
3 Cutting Force Model:

In this section the algorithm for modeling the cutting forces in voxelization framework is discussed. As mentioned before, the first step in calculating cutting forces during milling is to voxelize the work-piece and cutter. After that, the chosen tool path should be discretized into Cutter Locations (CL) and in this case of study the distance between two consecutive CL is equal to voxel size. For the next step, the engagement region between tool and work-piece should be determined. In discretized models like voxelization framework, only the contact zone of the tool and work-piece is taken into account. Voxelization framework, because of its discretized nature as well as its adoptability for being run on parallel frameworks, is one of the most powerful tools for detecting the contact zone of the cutter and work-piece. Based on a Boolean operation between each cutting and blank voxels, the contact zone for each Cutter Location (CL) can be detected. In voxelization framework each voxel can be assigned an integer number representing the location of the voxel with respect to a fixed coordinate system. Thus if the location number of the cutting voxel is equal to the location number of blank voxel, it means that the cutting process is happening and the cutting voxel of the tool is tolerating a force. Moreover as Figure 3 shows, in each CL the engaged cutting voxel and its local angle (θ) with respect to the tool origin can be selected and the start and exit angle can be calculated. These start and exit angles can be employed for studying the dynamics of machining, such as chattering during milling process.

![Figure 3: Cutting voxel position with respect to tool origin (Lazoglu, 2003)](image)

The tangential (F_t), radial (F_r) and axial (F_a) cutting forces acting on cutting voxels, located in cutting zone, for each CL are calculated as follows:

\[
\begin{align*}
F_t &= \text{Voxel Size} \times \left( K_{te} \sum_{i=1}^{n} C_{th}(\theta_i) \times \delta_i \right) + K_{te} \\
F_r &= \text{Voxel Size} \times \left( K_{re} \sum_{i=1}^{n} C_{th}(\theta_i) \times \delta_i \right) + K_{re} \\
F_a &= \text{Voxel Size} \times \left( K_{ae} \sum_{i=1}^{n} C_{th}(\theta_i) \times \delta_i \right) + K_{ae}
\end{align*}
\]
where $n$ represents the total number of the cutting voxels which are in the cutting zone in a specific CL. $C_\text{ub}(\theta)$ is the uncut chip thickness and is considered to be a function of the voxel position on the cutting helix. In the above equation, $\delta_i$ is a Boolean function which operates as follows:

$$
\delta_i = \begin{cases} 
1; & \text{when } \text{ith cutting voxel is in cutting zone} \\
0; & \text{when } \text{ith cutting voxel is not in cutting zone}
\end{cases}
$$

A dynamometer is able to measure forces in X, Y and Z directions with respect to CNC Machine coordinates so the tangential, radial and axial coordinate systems need to be transformed into X, Y and Z coordinate system by the transformation matrix as follows:

$$
\begin{bmatrix} 
F_x \\
F_y \\
F_z 
\end{bmatrix} = \begin{bmatrix} 
T_{\text{Transformation Matrix}} 
\end{bmatrix} \times \begin{bmatrix} 
F_x \\
F_y \\
F_z 
\end{bmatrix}
$$

The transformation matrix is a function of voxel position and for a generalized tool can be expressed as follows:

$$
T = \begin{bmatrix} 
\cos(\theta) & \sin(\theta)\times\sin(\kappa) & \sin(\theta)\times\cos(\kappa) \\
\sin(\theta) & \cos(\theta)\times\sin(\kappa) & \cos(\theta)\times\cos(\kappa) \\
0 & \cos(\kappa) & \sin(\kappa)
\end{bmatrix}
$$

where $\kappa$ for the voxel at point $P$, is its axial immersion as shown in figure 4:

Figure 4: Axial Immersion for Voxel at Point P (Engin & Altintas, 1999)

If the vibration of the tool can be neglected; the uncut chip thickness can be calculated based on the following equation:

$$
C_\text{ub}(\theta_i) = f_s \times \sin(\theta_i) \times \sin(\kappa)
$$

where $f_s$ is the feed per tooth and $\theta_i$ represents the angle which the $i$th cutting voxel in the cutting zone makes with the positive Y direction and $\kappa$ shows the axial immersion angle.

Sub-indices $c$ and $e$ in $K$ represents shear and edge coefficients and are calculated based on the mechanical tests (Engin & Altintas, 1999).

Cutting force prediction in voxelization framework can be applied for any arbitrary shape of the work-piece milled by generalized cutter, because there is no limitation on geometry for voxelizing either a tool or work-piece.
4 Simulation and Experimental Results:

In this section the simulation approach and experimental validation is studied. The algorithm was implemented on C++ codes but has been designed with a parallel implementation in mind and therefore has compatibility to be run in parallel on the GPU. The whole algorithm can be simplified as it has been shown in Figure 5.

Figure 5: The algorithm for calculating cutting force in voxel-model

A set of milling experiments were performed on a 3-axis Vertical CNC Machine in order to find the edge and shear coefficients and validate the simulation model. The cutter used was a two flute carbide flat-end mill with diameter of 6.35 millimeters, nominal helix angle of 30 degrees, with radial rake angle of 0 degrees. The calibration process was done on Kistler Type 9257B Dynamometer with Kistler amplifier Type 5017. MyDaq® acquisition board was employed to acquire data for Kistler Dynamometer at 20kHz sampling rate. The linear regression approach introduced by Schmitz and Smith was employed to approximate the edge and shear cutting coefficients (Schmitz & Smith, 2009).

The experiment was done on both aluminum and PMMA in order to show that the model works for both low and high forces. In both cases it shows the acceptable match between simulation and experimental results. The machining parameters for the experiment are shown in Table 2:

<table>
<thead>
<tr>
<th>PMMA Milling Experiment</th>
<th>Aluminum 6061 Milling Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of Cut</td>
<td>1.2 mm</td>
</tr>
<tr>
<td></td>
<td>Depth of Cut</td>
</tr>
<tr>
<td>Spindle Speed</td>
<td>1500 RPM</td>
</tr>
<tr>
<td></td>
<td>Spindle Speed</td>
</tr>
<tr>
<td>Feed-rate</td>
<td>762 mm/min</td>
</tr>
<tr>
<td></td>
<td>Feed-rate</td>
</tr>
</tbody>
</table>

Table 2: The cutting parameters for machining experiments

Figure 5 shows the comparison between the simulated models for cutting forces in X, Y and Z direction in voxelization framework and the experimented results. As the figure represents, there is a good agreement between the simulated model and experimental results.
5 Conclusion

A new method for cutting force prediction during 3-axis CNC machining was introduced in this paper. The method developed is based on the voxelization framework for digital manufacturing. Hence, different work-piece and cutter geometries can be easily implemented into the method. Instead of the whole tool geometry, only the cutting helix is voxelized in order to decrease the computational cost. The method has the ability to calculate the engagement zone between the cutter and work-piece.
based on the cutter location and geometries of the tool and work-piece. Moreover, the model can be adjusted to run on parallel framework which can significantly decrease the computational time. An analytical model based on the discretized space was introduced for calculating the cutting forces for each cutting voxel on the cutting helix. The results of the model validation experiment were also reported. Comparison of the simulated and measured data for the cutting forces in 3 directions showed a reasonable agreement.

References
Schmitz TL and Smith KS. Machining Dynamics; Frequency Response to Improved Productivity. New York: Springer-Verlag New York, LLC, 2009.